Name: ________________________ Class: ___________________ Date: __________ **ID: A**

Algebra II Practice Test 12.1 - 12.3

Multiple Choice
Identify the choice that best completes the statement or answers the question.

____ 1. Find the first 5 terms of the sequence with $a_1 = 6$ and $a_n = 2a_{n-1} - 1$ for $n \geq 2$.

a. 1, 2, 3, 4, 5
b. 6, 11, 21, 41, 81
c. 6, 12, 24, 48, 96
d. 6, 7, 8, 9, 10

____ 2. Find the first 5 terms of the sequence $a_n = 2^n - 5$.

a. –4, –1, 4, 11, 20
b. –3, –1, 3, 11, 27
c. 7, 9, 13, 21, 37
d. –3, –1, 1, 3, 5

____ 3. Write a possible explicit rule for *n*th term of the sequence 23.1, 20.2, 17.3, 14.4, 11.5, 8.6, ...

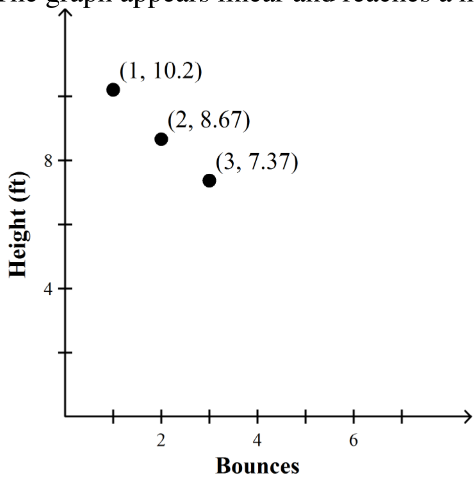
a. $a_n = 26 - 2.9n$
b. $a_n = 23.1 - 2.9n$
c. $a_n = 23.1(2.9^n)$
d. $a_n = 23.1(2.9n)$

Name: ________________________ ID: A

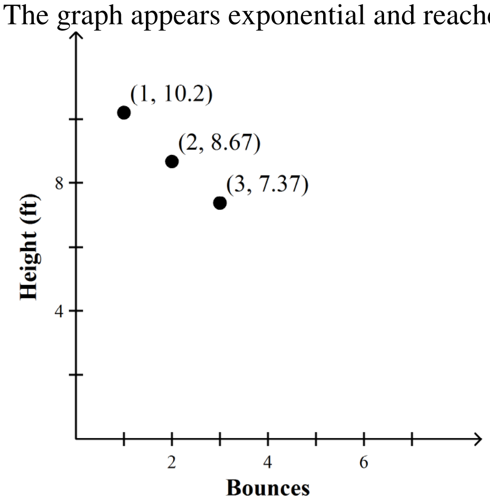
____ 4. A ball is dropped and bounces to a height of 10.2 feet. The ball rebounds to 85% of its previous height after each bounce. Graph the sequence and describe the pattern. How high is the ball on its 6th bounce?

Name: ________________________ **ID: A**

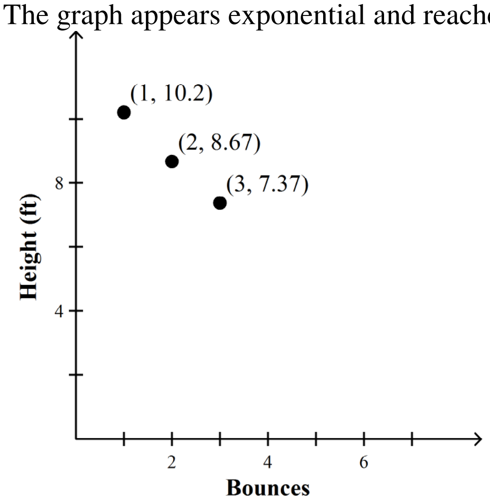
a. The graph appears linear and reaches a height of 6.9 ft on its 6th bounce.



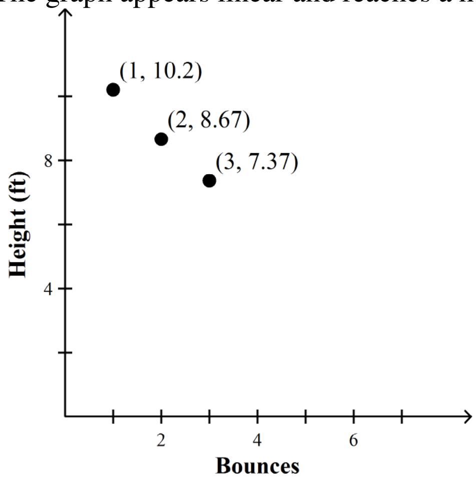
b. The graph appears exponential and reaches a height of 5.1 ft on its 6th bounce.



c. The graph appears exponential and reaches a height of 4.5 ft on its 6th bounce.

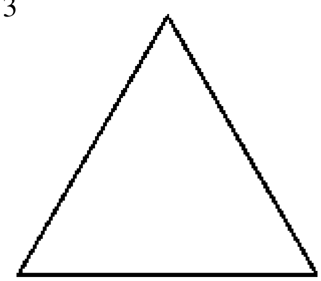


d. The graph appears linear and reaches a height of 7.5 ft on its 6th bounce.

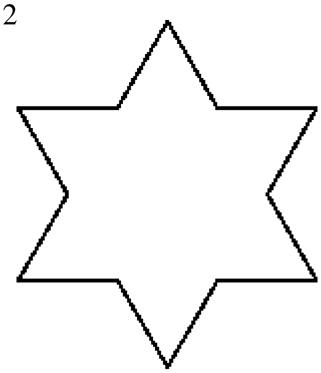


____ 5. The Von Koch snowflake is a fractal made by taking an equilateral triangle, removing the middle third of each side, and then building an equilateral triangle where each side was removed. Let a_i = the number of sides of figure *i*. Find the number of sides on the next 2 iterations.

$a_1 = 3$



$a_2 = 12$



a. $a_3 = 21$ and $a_4 = 30$
b. $a_3 = 48$ and $a_4 = 192$
c. $a_3 = 60$ and $a_4 = 360$
d. $a_3 = 27$ and $a_4 = 48$

____ 6. There are 3 squares of increasing size. The ratio of a side of one square to a side of the next larger square is $\frac{1}{\sqrt{2}}$. The first square has an area of 9 square units. Find the area of the third square.

a. $9\sqrt{2}$ square units
b. 18 square units
c. 36 square units
d. $81\sqrt{2}$ square units

____ 7. Write the series $-\frac{1}{2}+\frac{1}{4}-\frac{1}{6}+\frac{1}{8}-\frac{1}{10}+\frac{1}{12}$ in summation notation.

a. $\sum_{k=1}^{6}(-1)^k\left(\frac{1}{2k}\right)$
b. $\sum_{k=1}^{6}(-1)^{k+1}\left(\frac{1}{2k}\right)$
c. $\sum_{k=1}^{6}(-1)^k\left(\frac{1}{2}\right)k$
d. $\sum_{k=1}^{6}(-1)^{k+1}\left(\frac{1}{2}\right)k$

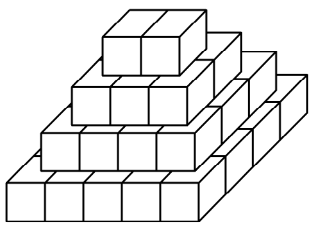
____ 8. Expand the series $\sum_{k=2}^{6}(-1)^k(7-k)k$ and evaluate.

a. 0 c. 50
b. 56 d. 6

____ 9. Evaluate the series $\sum_{k=1}^{22}k$.

a. 506 c. 22
b. 253 d. 23

____ 10. Alicia is building a block pyramid similar to the pyramid shown. The top level will always have two blocks. Alicia wants her pyramid to contain as many levels as possible. How many levels can her pyramid have if Alicia has 200 blocks?



a. 7 levels c. 9 levels
b. 8 levels d. 13 levels

____ 11. Determine whether the sequence –1, 7, 15, 23, 31, ... appears to be an arithmetic sequence. If so, find the common difference and the next three terms in the sequence.

a. Yes; common difference –8; next three terms are 23, 15, 7
b. Yes; common difference 7; next three terms are 38, 45, 52
c. Not an arithmetic sequence
d. Yes; common difference 8; next 3 terms are 39, 47, 55

____ 12. Find the 22nd term in the arithmetic sequence –5, –9, –13, –17, –21,...

a. –93 c. –110
b. –84 d. –89

____ 13. Find the missing terms in the arithmetic sequence 18, ___, ___, ___, 42.

a. 24, 36, 48 c. 6, 12, 18
b. 24, 30, 36 d. 48, 78, 108

____ 14. Find the 5th term of the arithmetic sequence with $a_7 = 25$ and $a_{13} = 55$.

a. 5 c. 15
b. 20 d. –5

____ 15. Find the sum for the arithmetic series $\sum_{k=1}^{13}15k-4$.

a. 13 c. 1313
b. 101 d. 2626

____ 16. A marching band formation consists of 8 rows. The first row has 5 musicians, the second has 7, the third has 9, and so on. How many musicians are in the last row?

a. 17 musicians
c. 21 musicians
b. 19 musicians
d. 96 musicians

____ 17. Write the arithmetic series $5 + 1 - 3 - 7 - 11 - 15 - 19$ in summation notation.

a. $\sum_{k=1}^{7} (5 - 4k)$
c. $\sum_{k=1}^{7} \left(5 - 4(k+1)\right)$
b. $\sum_{k=1}^{7} \left(9 - 4(k-1)\right)$
d. $\sum_{k=1}^{7} (9 - 4k)$

Algebra II Practice Test 12.1 - 12.3
Answer Section

MULTIPLE CHOICE

1. ANS: B

The first term is given $a_1 = 6$.
Substitute this value into the rule to find the next term.
Continue using each term to find the next term.

n	$2a_{n-1} - 1$	a_n
1		6
2	$2(6) - 1$	11
3	$2(11) - 1$	21
4	$2(21) - 1$	41
5	$2(41) - 1$	81

	Feedback
A	The first term is given. Use the rule to find each following term.
B	Correct!
C	Remember to use both parts of the rule when finding each of the terms.
D	You have the correct first term. Use the rule to find the following terms.

PTS: 1 DIF: Average REF: Page 862
OBJ: 12-1.1 Finding Terms of a Sequence by Using a Recursive Formula
NAT: 12.5.1.a TOP: 12-1 Introduction to Sequences

2. ANS: B

Make a table. Evaluate the sequence for $n = 1$ through $n = 5$.

n	$2^n - 5$	a_n
1	$2^1 - 5$	–3
2	$2^2 - 5$	–1
3	$2^3 - 5$	3
4	$2^4 - 5$	11
5	$2^5 - 5$	27

The first 5 terms are –3, –1, 3, 11, and 27.

	Feedback
A	You reversed the values of the power base and exponent.
B	Correct!
C	You added the two terms instead of subtracting.
D	You multiplied instead of raising to the power of *n*.

PTS: 1 DIF: Basic REF: Page 863
OBJ: 12-1.2 Finding Terms of a Sequence by Using an Explicit Formula
NAT: 12.5.1.a TOP: 12-1 Introduction to Sequences

3. ANS: A

Examine the differences.

	23.1	20.2	17.3	14.4	11.5	8.6
First difference		2.9	2.9	2.9	2.9	2.9

The first differences are constant, so the sequence is linear.

$a_n = 23.1 - 2.9(n - 1)$ The first term is 23.1, and each term is 2.9 less than the previous. Use $(n - 1)$ to get 23.1 when $n = 1$.

$a_n = 23.1 - 2.9n + 2.9$ Distribute and simplify.

$a_n = 26 - 2.9n$

	Feedback
A	Correct!
B	Adjust the rule so that when $n = 1$, the result is 23.1.
C	If the first differences are constant, the rule is linear and not exponential.
D	If the first differences are constant, the rule is linear. Add the multiples of the difference to the starting value.

PTS: 1 DIF: Average REF: Page 863 OBJ: 12-1.3 Writing Rules for Sequences
NAT: 12.5.1.b TOP: 12-1 Introduction to Sequences

4. ANS: C

Because the ball bounces to a height of 10.2 feet and then bounces to 85% of its previous height, the recursive rule is $a_1 = 10.2$ and $a_n = 0.85a_{n-1}$.

Use this rule to find other terms of the sequence.

$a_2 = 0.85a_1 = 0.85(10.2) = 8.67$

$a_3 = 0.85a_2 = 0.85(8.67) = 7.37$

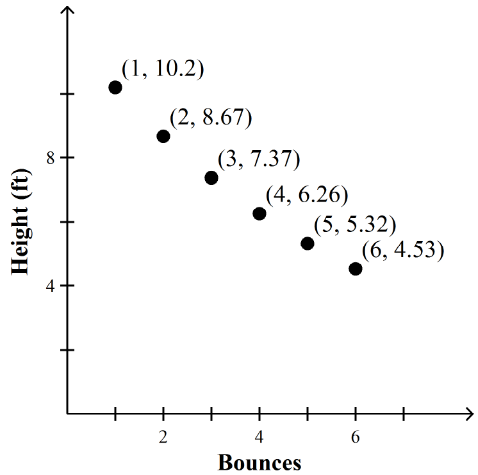
The graph appears to be exponential.
Use the pattern to write the explicit rule.

$a_n = (10.2)0.85^{n-1}$

Use this rule to find the bounce height on the 6th bounce.

$a_6 = (10.2)0.85^5 = 4.5$

The ball is approximately 4.5 feet high on the 6th bounce.

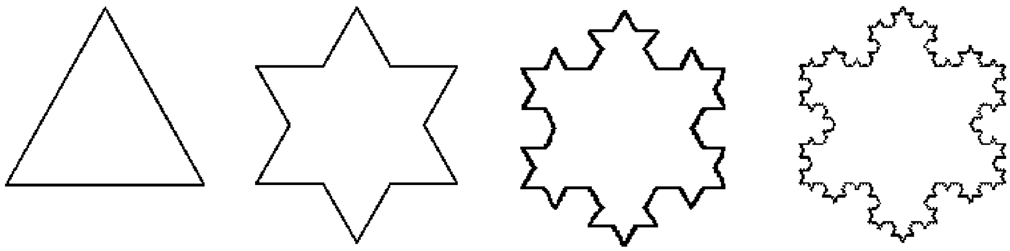


	Feedback
A	Use the recursive rule to find terms in the sequence.
B	Use the recursive rule to find terms in the sequence.
C	Correct!
D	Multiply the height after a bounce by 0.85 to get the height of the next bounce.

PTS: 1 DIF: Average REF: Page 864 OBJ: 12-1.4 Application
NAT: 12.5.1.a TOP: 12-1 Introduction to Sequences

5. ANS: B

Here are the first 4 iterations.



Each iteration multiplies the number of sides by 4. The number of sides can be represented by the equation $a_n = 3(4)^{n = 1}$.

Use the equation to find a_3 and a_4.

$a_3 = 3(4)^{3 = 1} = 3(4)^2 = 3(16) = 48$

$a_4 = 3(4)^{4 = 1} = 3(4)^3 = 3(64) = 192$

The next two iterations result in 48 and 192 sides.

	Feedback
A	After the iteration, each side is replaced by 4 smaller sides.
B	Correct!
C	After the iteration, each side is replaced by 4 smaller sides.
D	After the iteration, each side is replaced by 4 smaller sides.

PTS: 1 DIF: Average REF: Page 864 OBJ: 12-1.5 Iteration of Fractals
TOP: 12-1 Introduction to Sequences

6. ANS: C

Step 1 Find the length of the sides of the third square.

The sides of the first square are 3 units long. The sides of the second square are $3\sqrt{2}$ units long. The sides of the third square are $3\sqrt{2} \cdot \sqrt{2} = 6$ units long.

Step 2 Find the area of the third square.

$A = 6 \times 6 = 36$ square units

	Feedback
A	The sides of the third square are 6 units long.
B	The sides of the third square are 6 units long.
C	Correct!
D	The sides of the third square are 6 units long.

PTS: 1 DIF: Advanced NAT: 12.5.1.a TOP: 12-1 Introduction to Sequences

7. ANS: A

Find a rule for the *k*th term.

$a_k = (-1)^k \left(\dfrac{1}{2k} \right)$ Explicit formula

Write the notation for the first 6 items.

$\sum_{k=1}^{6} (-1)^k \left(\dfrac{1}{2k} \right)$ Summation notation

	Feedback
A	Correct!
B	Does the sequence start with a minus sign or a plus sign?
C	Is the whole fraction multiplied by *k* or just the denominator?
D	Does the sequence start with a minus sign or a plus sign? Is the whole fraction multiplied by *k* or just the denominator?

PTS: 1 DIF: Average REF: Page 870 OBJ: 12-2.1 Using Summation Notation
TOP: 12-2 Series and Summation Notation

8. ANS: D

Expand the series by replacing *k*. Then evaluate the sum.

$$\sum_{k=2}^{6} (-1)^k (7k - k^2)$$

$$= (-1)^2((7)(2) - 2^2) + (-1)^3((7)(3) - 3^2)$$

$$+(-1)^4((7)(4) - 4^2) + (-1)^5((7)(5) - 5^2) + (-1)^6((7)(6) - 6^2)$$

$$= 10 - 12 + 12 - 10 + 6$$

$$= 6$$

	Feedback
A	The lower limit for *k* is not 1.
B	Expand the series by replacing *k*. Then evaluate the sum.
C	When k is odd, (–1)^ *k* is negative.
D	Correct!

PTS: 1 DIF: Average REF: Page 871 OBJ: 12-2.2 Evaluating a Series
TOP: 12-2 Series and Summation Notation

9. ANS: B

$$\sum_{k=1}^{22} k = \frac{n(n+1)}{2} = \frac{22(23)}{2} = 253$$

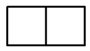
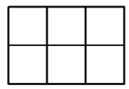
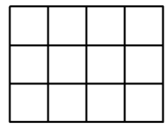
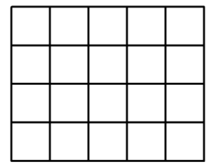
Use the summation formula for a linear series.

	Feedback
A	Use the correct summation formula.
B	Correct!
C	Find the sum of the first n natural numbers, where n is the number on top of the summation notation.
D	Use the summation formula for a linear series.

PTS: 1 DIF: Average REF: Page 872 OBJ: 12-2.3 Using Summation Formulas
TOP: 12-2 Series and Summation Notation

10. ANS: A

Make a table and a diagram. Number the levels starting from the top level.

Level	1	2	3	4
Diagram				
Blocks	$1 \cdot 2 = 2$	$2 \cdot 3 = 6$	$3 \cdot 4 = 12$	$4 \cdot 5 = 20$

The number of blocks in each level is $k(k+1)$ where k is the level number. Write a series to represent the total number of blocks in n rows.

$\sum_{k=1}^{n} k(k+1)$, where k is the level number and n is the total number of levels.

Evaluate the series for several n-values.

$$\sum_{k=1}^{6} k(k+1) = 1(2) + 2(3) + 3(4) + 4(5) + 5(6) + 6(7) = 112$$

$$\sum_{k=1}^{7} k(k+1) = 1(2) + 2(3) + 3(4) + 4(5) + 5(6) + 6(7) + 7(8) = 168$$

$$\sum_{k=1}^{8} k(k+1) = 1(2) + 2(3) + 3(4) + 4(5) + 5(6) + 6(7) + 7(8) + 8(9) = 240$$

Because Alicia has 200 blocks, the pyramid can have at most 7 levels.

	Feedback
A	Correct!
B	Starting from the top level, the number of blocks in each level is $k(k + 1)$. Evaluate the series SUM $k(k + 1)$ from $k = 1$ to n for several values of n.
C	Starting from the top level, the number of blocks in each level is $k(k + 1)$. Evaluate the series SUM $k(k + 1)$ from $k = 1$ to n for several values of n.
D	Starting from the top level, the number of blocks in each level is $k(k + 1)$. Evaluate the series SUM $k(k + 1)$ from $k = 1$ to n for several values of n.

PTS: 1 DIF: Average REF: Page 872

OBJ: 12-2.4 Problem-Solving Application

TOP: 12-2 Series and Summation Notation

11. ANS: D
For a sequence to be an arithmetic sequence, each number subtracted from the one before it should result in a common difference.
This sequence is arithmetic. Each term differs from the previous one by 8.

	Feedback
A	Find the difference between each term and the one before it. If this difference is always the same, the sequence is arithmetic.
B	Find the difference between each term and the one before it. If this difference is always the same, the sequence is arithmetic.
C	Find the difference between each term and the one before it. If this difference is always the same, the sequence is arithmetic.
D	Correct!

PTS: 1 DIF: Basic REF: Page 879
OBJ: 12-3.1 Identifying Arithmetic Sequences NAT: 12.5.1.a
TOP: 12-3 Arithmetic Sequences and Series KEY: arithmetic sequence

12. ANS: D
Find a specific term from a given sequence by using the equation $a_n = a_1 + (n-1)d$, where:
a_n = your result
a_1 = the initial term of the sequence
n = the number in the sequence you want to calculate
d = the common difference between the terms

n is given in the problem, a_1 is the first term in the sequence, and d is the difference between adjacent terms.

	Feedback
A	Find the correct term in the sequence.
B	The equation for the nth term contains $(n - 1)$.
C	Check that the common difference is correct.
D	Correct!

PTS: 1 DIF: Average REF: Page 880
OBJ: 12-3.2 Finding the nth Term Given an Arithmetic Sequence
NAT: 12.5.1.a TOP: 12-3 Arithmetic Sequences and Series
KEY: arithmetic sequence | finding given term | nth term

13. ANS: B

Step 1 Find the common difference.

$a_n = a_1 + (n-1)d$	General rule
$42 = 18 + (5-1)d$	Substitute 42 for a_n, 18 for a_1, and 5 for n.
$6 = d$	Solve for d.

Step 2 Find the missing terms using $d = 6$ and $a_1 = 18$.

$a_2 = 18 + (2-1)(6)$ $a_2 = 24$	$a_3 = 18 + (3-1)(6)$ $a_3 = 30$	$a_4 = 18 + (4-1)(6)$ $a_4 = 36$

The missing terms are 24, 30, and 36.

	Feedback
A	Use the general rule to find the common difference. Then use the general rule, first term, and common difference to find the missing terms.
B	Correct!
C	Use the general rule to find the common difference. Then use the general rule, first term, and common difference to find the missing terms.
D	Use the general rule to find the common difference. Then use the general rule, first term, and common difference to find the missing terms.

PTS: 1 DIF: Average REF: Page 881 OBJ: 12-3.3 Finding Missing Terms
NAT: 12.5.1.a TOP: 12-3 Arithmetic Sequences and Series

14. ANS: C

Step 1 Find the common difference.
$a_{13} = a_7 + (13 - 7)d$
$55 = 25 + (6)d$
$5 = d$

Step 2 Find a_1.
$a_7 = a_1 + (7 - 1)d$
$25 = a_1 + (6)5$
$-5 = a_1$

Step 3 Write a rule for the sequence and evaluate to find a_5.
$a_n = a_1 + (n - 1)d$
$a_5 = -5 + (5 - 1)5$
$a_5 = 15$

	Feedback
A	This is the common difference, now find the value of the requested term.
B	When finding the nth term, multiply the difference by(n – 1) and then add the first term.
C	Correct!
D	This is the first term, now find the value of the requested term.

PTS: 1 DIF: Average REF: Page 881
OBJ: 12-3.4 Finding the nth Term Given Two Terms NAT: 12.5.1.a
TOP: 12-3 Arithmetic Sequences and Series

15. ANS: C

Find the 1st and 13th terms.

$a_1 = 15(1) - 4 = 11$

$a_{13} = 15(13) - 4 = 191$

Find S_{13}.

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

$$S_{13} = 13\left(\frac{a_1 + a_{13}}{2}\right)$$

$$S_{13} = 13\left(\frac{11 + 191}{2}\right)$$

$$S_{13} = 1313$$

	Feedback
A	This is the number of terms in the sum. Now find the sum.
B	Add the first and last term, multiply by the number of terms and divide by 2.
C	Correct!
D	Add the first and last term, multiply by the number of terms and divide by 2.

PTS: 1 DIF: Average REF: Page 882
OBJ: 12-3.5 Finding the Sum of an Arithmetic Series
TOP: 12-3 Arithmetic Sequences and Series

16. ANS: B

Write a general rule using $a_1 = 5$ and $d = 2$.

$a_n = a_1 + (n-1)d$	Explicit rule for the *n*th term
$a_8 = 5 + (8-1)2$	Substitute.
$a_8 = 19$	Simplify.

There are 19 musicians in the last row.

	Feedback
A	List the number of musicians in each row, and check your answer.
B	Correct!
C	To find the 8th term, add the product of 7 and the common difference to the first term.
D	List the number of musicians in each row, and check your answer.

PTS: 1 DIF: Average REF: Page 883 OBJ: 12-3.6 Application
TOP: 12-3 Arithmetic Sequences and Series

17. ANS: D

The general form for the *k*th term in an arithmetic series is $a_k = a_0 + d(k-1)$ where a_0 is the first term d is the common difference between consecutive terms. In the question, a_0 = 5 and d = –4. Therefore,

$$\begin{aligned} a_k &= a_0 + d(k-1) \\ &= 5 - 4(k-1) \\ &= 9 - 4k \end{aligned}$$

The general form for a partial arithmetic sum is $\sum_{k=1}^{n} a_k$ where n is the total number of terms in the series.

Therefore, $\sum_{k=1}^{n} a_k = \sum_{k=1}^{7} (9-4k)$.

	Feedback
A	The *k*-th term in an arithmetic series is the first term plus (k – 1) times the common difference.
B	The *k*-th term in an arithmetic series is the first term plus (k – 1) times the common difference.
C	The *k*-th term in an arithmetic series is the first term plus (k – 1) times the common difference.
D	Correct!

PTS: 1 DIF: Advanced TOP: 12-3 Arithmetic Sequences and Series